Ejercicios de la regla de L'Hôpital

1 $\lim_{x \to 0} \frac{e^x - e^{-x}}{\operatorname{sen} x}$

2 $\lim_{x \to \infty^+} \frac{x}{(\ln x)^3 + 2x}$

3 $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x} \right)$

4 $\lim_{x \to \frac{\pi}{4}} (\operatorname{tg} x - 1) \sec 2x$

5 $\lim_{x \to 0} x^{\operatorname{tg} x}$

6 $\lim_{x \to 0} (\operatorname{cotg} x)^{\operatorname{sen} x}$

7 $\lim_{x \to 2} \left(\frac{x}{2} \right)^{\frac{1}{x-2}}$

8 $\lim_{x \to 1} \frac{\ln x}{x - 1}$

9 $\lim_{x \to 0} \frac{\operatorname{sen} 3x}{x - \frac{3}{2} \operatorname{sen} 2x}$

10 $\lim_{x \to 0} (\operatorname{arc\,sen} x \ \operatorname{cot} g\, x)$

11 $\lim_{x \to 0} \left[\frac{1}{2} \frac{\operatorname{sen} x}{\operatorname{tg} x} (1 + \operatorname{tg} 2x)^{\frac{4}{x}} \right]$

12 $\lim_{x \to 0} \frac{\ln(1 + x) - \operatorname{sen} x}{x \operatorname{sen} x}$

13 $\lim_{x \to 0} \frac{1 + \operatorname{sen} x - e^x}{(\operatorname{arc\,tg} x)^2}$

14 $\lim_{x \to 0} \left[\frac{1}{\ln(1 + x)} - \frac{1}{x} \right]$

15 $\lim_{x\to 0}\left(\frac{1+tgx}{1+senx}\right)^{\frac{1}{senx}}$

16 $\lim_{x\to 0} x^{senx}$

Solución ejercicios de la regla de L'Hôpital

1

$$\lim_{x\to 0}\frac{e^x - e^{-x}}{sen\,x}$$

$$\lim_{x\to 0}\frac{e^x - e^{-x}}{sen\,x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{e^x - e^{-x}}{sen\,x} = \lim_{x\to 0}\frac{e^x + e^{-x}}{\cos x} = 2$$

2

$$\lim_{x\to\infty^+}\frac{x}{(\ln x)^3 + 2x}$$

$$\lim_{x\to\infty^+}\frac{x}{(\ln x)^3 + 2x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty^+}\frac{x}{(\ln x)^3 + 2x} = \lim_{x\to\infty^+}\frac{1}{3(\ln x)^2\frac{1}{x} + 2}$$

3

$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{sen\,x}\right)$$

$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{sen\,x}\right) = \infty - \infty$$

$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{sen\,x}\right) = \lim_{x\to 0}\frac{senx - x}{x\,senx} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{senx - x}{x\,senx} = \lim_{x\to 0}\frac{\cos x - 1}{senx + x\cos x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\cos x - 1}{senx + x\cos x} = \lim_{x\to 0}\frac{-senx}{\cos x + \cos x - x\,senx} = 0$$

4

$$\lim_{x\to\frac{\pi}{4}}(tg\,x-1)\sec 2x$$

$$\lim_{x\to\frac{\pi}{4}}(tg\,x-1)\sec 2x = 0\cdot\infty$$

$$\lim_{x\to\frac{\pi}{4}}\frac{tg\,x-1}{\cos 2x} = \frac{0}{0}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{tg\,x-1}{\cos 2x} = \frac{1+tg^2x}{-2\,sen2x} = -1$$

5

$$\lim_{x\to 0}x^{tg\,x}$$

$A = x^{tg\,x}$ $\qquad ln\,A = tg\,x\ln x$ $\qquad A = e^{tg\,x\ln x}$

$$\lim_{x\to 0}x^{tg\,x} = \lim_{x\to 0}e^{tg\,x\ln x} = e^{\lim_{x\to 0}\left(e^{tg\,x\ln x}\right)} = e^{\lim_{x\to 0}\frac{\ln x}{cotg\,x}} = e^{\lim_{x\to 0}\left(-\frac{\frac{1}{x}}{cosec^2x}\right)} =$$

$$= e^{\lim_{x\to 0}\frac{-sen^2x}{x}} = e^{\lim_{x\to 0}(-2\,sen\,x\cos x)} = e^0 = 1$$

6

$$\lim_{x\to 0}(cotg\,x)^{sen\,x}$$

$A = (cotg\,x)^{sen\,x}$ $\qquad ln\,A = sen\,x\,ln(cotg\,x)$

$$A = e^{sen\,x\,ln(cotg\,x)}$$

$$\lim_{x\to 0}(cotg\,x)^{sen\,x} = \lim_{x\to 0}e^{sen\,x\,ln(cotg\,x)} = e^{\lim_{x\to 0}\frac{ln(cotg\,x)}{cosec\,x}} =$$

$$e^{\lim_{x\to 0}\frac{\frac{-cosec^2x}{cotg\,x}}{\frac{-\cos x}{sen^2x}}} = e^{\lim_{x\to 0}\frac{cosec^2x\,sen^2x}{\cos x\,cotg\,x}} = e^{\lim_{x\to 0}\frac{1}{\cos x\,\frac{\cos x}{sen\,x}}} = e^{\lim_{x\to 0}\frac{sen\,x}{\cos^2x}} = e^0 = 1$$

7

$$\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}}$$

$$\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}} = 1^{\infty} \qquad A = \left(\frac{x}{2}\right)^{\frac{1}{x-2}}$$

$$\ln A = \frac{1}{x-2}\ln\left(\frac{x}{2}\right) \qquad A = e^{\frac{1}{x-2}\ln\left(\frac{x}{2}\right)}$$

$$\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}} = e^{\lim\limits_{x\to 2}\left[\frac{1}{x-2}\ln\left(\frac{x}{2}\right)\right]} = e^{\lim\limits_{x\to 2}\left[\frac{\ln\left(\frac{x}{2}\right)}{x-2}\right]} =$$

$$e^{\lim\limits_{x\to 2}\frac{\frac{1}{2}}{\frac{x}{2}}} = e^{\frac{1}{2}} = \sqrt{e}$$

8

$$\lim_{x\to 1}\frac{\ln x}{x-1}$$

$$\lim_{x\to 1}\frac{\ln x}{x-1} = \frac{0}{0}$$

$$\lim_{x\to 1}\frac{\ln x}{x-1} = \lim_{x\to 1}\frac{1}{x} = 1$$

9

$$\lim_{x\to 0}\frac{\operatorname{sen}3x}{x-\frac{3}{2}\operatorname{sen}2x}$$

$$\lim_{x\to 0}\frac{\operatorname{sen}3x}{x-\frac{3}{2}\operatorname{sen}2x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\operatorname{sen}3x}{x-\frac{3}{2}\operatorname{sen}2x} = \lim_{x\to 0}\frac{3\cos 3x}{1-3\cos 2x} = -\frac{3}{2}$$

10

$$\lim_{x\to 0}\left(arc\ sen\,x\ \cot g\,x\right)$$

$$\lim_{x\to 0}\left(arc\ sen\,x\ \cot g\,x\right) = 0\cdot\infty$$

$$\lim_{x\to 0}\left(arc\ sen\,x\ \cot g\,x\right) = \lim_{x\to 0}\frac{\cos x\ arc\ sen\,x}{sen\,x} =$$

$$= \lim_{x\to 0} \frac{-sen\,x\,arc\ sen\,x + \dfrac{\cos x}{\sqrt{1-x^2}}}{\cos x} = 1$$

11

$$\lim_{x\to 0}\left[\frac{1}{2}\frac{sen\,x}{tg\,x}(1+tg\,2x)^{\frac{4}{x}}\right]$$

$$\lim_{x\to 0}\left[\frac{1}{2}\frac{sen\,x}{\dfrac{senx}{\cos x}}(1+tg\,2x)^{\frac{4}{x}}\right] = \frac{1}{2}\lim_{x\to 0}(\cos x)\lim_{x\to 0}(1+tg\,2x)^{\frac{4}{x}} =$$

$$= \frac{1}{2}\lim_{x\to 0}(1+tg\,2x)^{\frac{4}{x}} = 1^{\infty}$$

$$A = (1+tg\,2x)^{\frac{4}{x}} \qquad \ln A = \frac{4}{x}\ln(1+tg\,2x) \qquad A = e^{\frac{4}{x}\ln(1+tg\,2x)}$$

$$\frac{1}{2}\lim_{x\to 0}(1+tg\,2x)^{\frac{4}{x}} = \frac{1}{2}e^{\lim\limits_{x\to 0}\frac{4\ln(1+tg\,2x)}{x}} = \frac{1}{2}e^{\lim\limits_{x\to 0}\frac{4\cdot 2(1+tg^2 2x)}{\ln(1+tg\,2x)}} = \frac{1}{2}e^{8}$$

12

$$\lim_{x\to 0}\frac{\ln(1+x) - senx}{x\,senx}$$

$$\lim_{x\to 0}\frac{\ln(1+x) - senx}{x\,senx} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\ln(1+x) - senx}{x\,senx} = \lim_{x\to 0}\frac{\dfrac{1}{1+x} - \cos x}{senx + x\cos x} =$$

$$= \lim_{x\to 0}\frac{-\dfrac{1}{(1+x)^2} + senx}{\cos x + \cos x - xsenx} = -\frac{1}{2}$$

13

$$\lim_{x\to 0}\frac{1+senx - e^{x}}{(arc\ tgx)^2}$$

$$\lim_{x\to 0}\frac{1+senx - e^{x}}{(arc\ tgx)^2} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{1+\operatorname{sen}x-e^{x}}{(\operatorname{arc\,tg}x)^{2}}=\lim_{x\to 0}\frac{\cos x-e^{x}}{\frac{2\operatorname{arc\,tg}x}{1+x^{2}}}=$$

$$=\lim_{x\to 0}\frac{\operatorname{sen}x-e^{x}}{\frac{2-4x\operatorname{arc\,tg}x}{(1+x^{2})^{2}}}=-\frac{1}{2}$$

14

$$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]$$

$$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]=\infty-\infty$$

$$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]=\lim_{x\to 0}\frac{x-\ln(1+x)}{x\ln(1+x)}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{x-\ln(1+x)}{x\ln(1+x)}=\lim_{x\to 0}\frac{1-\frac{1}{1+x}}{\ln(1+x)+\frac{x}{1+x}}=$$

$$\lim_{x\to 0}\frac{\frac{1+x-1}{1+x}}{\frac{(1+x)\ln(1+x)+x}{1+x}}=\lim_{x\to 0}\frac{x}{\ln(1+x)+x\ln(1+x)+x}=$$

$$=\lim_{x\to 0}\frac{1}{\frac{1}{1+x}+\ln(1+x)+\frac{x}{1+x}+1}=\frac{1}{2}$$

15

$$\lim_{x\to 0}\left(\frac{1+tgx}{1+senx}\right)^{\frac{1}{senx}}$$

$$\lim_{x\to 0}\left(\frac{1+tgx}{1+senx}\right)^{\frac{1}{senx}}=1^{\infty}$$

$$A=\left(\frac{1+tgx}{1+senx}\right)^{\frac{1}{senx}}\qquad \ln A=\frac{1}{senx}\ln\left(\frac{1+tgx}{1+senx}\right)$$

Aplicando las **propiedades de los logaritmos** en el segundo miembro tenemos:

$$\ln A = \frac{1}{senx}\left[\ln(1+tgx) - \ln(1+senx)\right]$$

$$\ln A = \frac{\ln(1+tgx) - \ln(1+senx)}{senx}$$

$$A = e^{\frac{\ln(1+tgx)-\ln(1+senx)}{senx}}$$

$$\lim_{x\to 0}\left(\frac{1+tgx}{1+senx}\right)^{\frac{1}{senx}} = e^{\lim\limits_{x\to 0}\frac{\ln(1+tgx)-\ln(1+senx)}{senx}} =$$

$$= e^{\lim\limits_{x\to 0}\frac{\frac{1+tg^2x}{1+tgx}-\frac{\cos x}{1+senx}}{\cos x}} = e^0 = 1$$

16

$$\lim_{x\to 0} x^{senx}$$

$$\lim_{x\to 0} x^{senx} = 0^0$$

$$A = x^{senx} \qquad \ln A = senx \ln x \qquad A = e^{senx\ln x}$$

$$\lim_{x\to 0} x^{senx} = e^{\lim\limits_{x\to 0}(senx\ln x)} = e^{\lim\limits_{x\to 0}\frac{\ln x}{\frac{1}{senx}}} = e^{\lim\limits_{x\to 0}\frac{\frac{1}{x}}{\frac{-\cos x}{sen^2 x}}} =$$

$$e^{\lim\limits_{x\to 0}\frac{-sen^2x}{x\cos x}} = e^{\lim\limits_{x\to 0}\frac{-2senx\cos x}{\cos x - xsenx}} = e^0 = 1$$